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THERMAL RADIATION AND CHEMICAL REACTION EFFECTS ON EXPONENTIALLY ACCELERATED VERTICAL PLATE WITH VARIABLE TEMPERATURE AND UNIFORM MASS DIFFUSION

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ABSTRACT

Theoretical solution of thermal radiation effects on unsteady flow past an exponentially accelerated vertical plate with variable temperature and uniform mass diffusion has been studied. The plate temperature as well as concentration level near the plate are raised uniformly. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The effect of velocity profiles are studied for different physical parameters like thermal radiation parameter, thermal Grashof number, mass Grashof number and Schmidt number. It is observed that the velocity increases with increasing values the thermal Grashof number or mass Grashof number. The trend is just reversed with respect to the thermal radiation parameter.

KEYWORDS: exponential, chemical reaction, radiation, vertical plate, heat and mass transfer.

INTRODUCTION

Radiative heat and mass transfer play an important role in manufacturing industries for the design of fins, steel rolling, nuclear power plants, gas turbines and various propulsion device for aircraft, combustion and furnace design, materials processing, energy utilization, temperature measurements, remote sensing for astronomy and space exploration, food processing and cryogenic engineering, as well as numerous agricultural, health and military applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the combined effect of thermal radiation and mass diffusion.

Natural convection on flow past an linearly accelerated vertical plate in the presence of viscous dissipative heat using perturbation method by Gupta *et al* (1979). Kafousias and Raptis (1981) extended this problem to include mass transfer effects subjected to variable suction or injection. Soundalgekar (1982) studied the mass transfer effects on flow past a uniformly accelerated vertical plate. Mass transfer effects on flow past an accelerated vertical plate with uniform heat flux was analyzed by Singh and Singh (1983). Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar (1984). The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo (1986). Mass transfer effects on exponentially accelerated infinite vertical plate with constant heat flux and uniform mass diffusion was studied by Basant Kumar Jha *et al* (1991). Agrawal et al (1998) studied free convection due to thermal and mass diffusion in laminar flow of an accelerated infinite vertical plate in the presence of magnetic filed. Agrawal et al (1999) further extended the problem of unsteady free convective flow and mass diffusion of an electrically conducting elasto-viscous fluid past a parabolic starting motion of the infinite vertical plate with transverse magnetic plate. The governing equations are tackled using Laplace transform technique.

It is proposed to study the effects of on flow past an infinite vertical plate subjected to exponential motion with variable temperature and uniform mass diffusion, in the presence of thermal radiation. The solutions are in terms of exponential and complementary error function.

MATHEMATICAL ANALYSIS

Here the unsteady flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature and uniform diffusion, in the presence of thermal radiation has been considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The x'-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ and concentration C'_∞. At time $t' > 0$, the plate is started with a velocity $u = u_0 \exp(a't')$ in its own plane against gravitational field and the temperature from the plate is raised to T_w and the concentration level near the plate are also raised to C'_w. The plate is infinite in length all the terms in the governing equations will be independent of x' and there is no flow along y-direction. Then under usual Boussinesq's approximation for unsteady parabolic starting motion is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} - K_l(C' - C'_\infty) \tag{3}$$

With the following initial and boundary conditions:

$$u = 0, \qquad T = T_\infty, \qquad C' = C'_\infty \quad \text{for all } y, t' \leq 0$$

$$t' > 0: \quad u = u_0 \exp(a't'), \quad T = T_\infty + (T_w - T_\infty)At' \quad C' = C'_w \quad \text{at } y = 0 \tag{4}$$
$$u \to 0 \qquad T \to T_\infty, \qquad C' \to C'_\infty \quad \text{as } y \to \infty$$

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \sigma(T_\infty^4 - T^4) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho \, C_p \, \frac{\partial T}{\partial t'} = k \, \frac{\partial^2 T}{\partial y^2} + 16 a^* \sigma T_\infty^3 \, (T_\infty - T) \tag{7}$$

On introducing the following non-dimensional quantities:

$$U = u\left(\frac{u_0}{v^2}\right)^{\!1/3}, \quad t = \left(\frac{u_0^{\,2}}{v}\right)^{\!1/3} t', \quad Y = y\left(\frac{u_0}{v^2}\right)^{\!1/3}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C'_\infty}{C'_w - C'_\infty}$$

$$Gr = \frac{g\beta(T - T_\infty)}{(v.u_0)^{1/3}}, \quad Gc = \frac{g\beta(C' - C'_\infty)}{(v.u_0)^{1/3}}, \quad R = \frac{16 a^* \sigma . T_\infty^{\,3}}{k}\left(\frac{v^2}{u_0}\right)^{\!\frac{2}{3}}, \quad \Pr = \frac{\mu C_p}{k}, \quad Sc = \frac{v}{D} \tag{8}$$

in equations (1), (3) and (7), reduces to

$$\frac{\partial U}{\partial t} \;=\; Gr\theta + GcC + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} \;=\; \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{\Pr}\theta \tag{10}$$

$$\frac{\partial C}{\partial t} \;=\; \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} - KC \tag{11}$$

The initial and boundary conditions in non-dimensional quantities are
$$U = 0, \qquad \theta = 0, \quad C = 0, \quad \text{for all} \quad Y, t \le 0$$
$$t > 0: \quad U = \exp(at), \quad \theta = t, \quad C = 1, \quad \text{at} \qquad Y = 0$$
$$U = 0, \qquad \theta \to 0, \quad C \to 0 \quad \text{as} \qquad Y \to \infty \tag{12}$$

The dimensionless governing equations (6) to (8) and the corresponding initial and boundary conditions (12) are tackled using Laplace transform technique.

$$\theta = \frac{t}{2}\Big[\exp(2\eta\sqrt{bt\Pr})\,erfc(\eta\sqrt{\Pr} + \sqrt{bt}) + \exp(-2\eta\sqrt{bt\Pr})\,erfc(\eta\sqrt{\Pr} - \sqrt{bt})\Big]$$
$$-\frac{\eta\sqrt{\Pr t}}{2\sqrt{b}}\Big[\exp(-2\eta\sqrt{bt\Pr})\,erfc(\eta\sqrt{\Pr} - \sqrt{bt}) - \exp(2\eta\sqrt{bt\Pr})\,erfc(\eta\sqrt{\Pr} + \sqrt{bt})\Big] \tag{13}$$

$$C = \frac{1}{2}\Big[\exp(2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc} + \sqrt{Kt}) + \exp(-2\eta\sqrt{KtSc})\,erfc(\eta\sqrt{Sc} - \sqrt{Kt})\Big] \tag{14}$$

$$U = \frac{\exp(at)}{2}\Big[\exp(2\eta\sqrt{at})\,erfc(\eta + \sqrt{at}) + \exp(-2\eta\sqrt{at})\,erfc(\eta - \sqrt{at})\Big]$$
$$+\left[\frac{Gr}{c^2(1-\Pr)} + \frac{Gc}{d^2(1-Sc)}\right]erfc(\eta)$$

$$+ \frac{Gr\,t}{c(1-\mathrm{Pr})}\left[(1+2\eta^2)erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\exp(-\eta^2)\right]$$

$$- \frac{Gr}{c^2(1-\mathrm{Pr})}\left\{\frac{\exp(ct)}{2}\left[\exp(2\eta\sqrt{ct})\,erfc(\eta+\sqrt{ct}) + \exp(-2\eta\sqrt{ct})\,erfc(\eta-\sqrt{ct})\right]\right\}$$

$$- \frac{Gc}{d(1-Sc)}\left\{\frac{\exp(dt)}{2}\left[\exp(2\eta\sqrt{dt})\,erfc(\eta+\sqrt{dt}) + \exp(-2\eta\sqrt{dt})\,erfc(\eta-\sqrt{dt})\right]\right\}$$

$$- \frac{Gr(1+ct)}{2c^2(1-\mathrm{Pr})}\left\{\left[\begin{array}{l}\exp\left(2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\mathrm{Pr}}+\sqrt{bt}\right)\\ +\exp\left(-2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\mathrm{Pr}}-\sqrt{bt}\right)\end{array}\right]\right\}$$

$$+ \frac{\mathrm{Pr}\,Gr\,\eta\sqrt{t}}{2c(1-\mathrm{Pr})\sqrt{R}}\left\{\left[\exp\left(-2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\mathrm{Pr}}-\sqrt{bt}\right) - \exp\left(2\eta\sqrt{Rt}\right)erfc\left(\eta\sqrt{\mathrm{Pr}}+\sqrt{bt}\right)\right]\right\}$$

$$- \frac{Gc}{d(1-Sc)}\left\{\left(\frac{1}{2}\right)\left[\exp\left(2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}+\sqrt{Kt}\right) + \exp\left(-2\eta\sqrt{Kt\,Sc}\right)erfc\left(\eta\sqrt{Sc}-\sqrt{Kt}\right)\right]\right\}$$

$$+ \frac{Gc}{d(1-Sc)}\left\{\frac{\exp(dt)}{2}\left[\begin{array}{l}\exp\left(2\eta\sqrt{Sc(K+d)t}\right).erfc\left(\eta\sqrt{Sc}+\sqrt{(K+d)t}\right)\\ +\exp\left(-2\eta\sqrt{Sc(K+d)t}\right).erfc\left(\eta\sqrt{Sc}-\sqrt{(K+d)t}\right)\end{array}\right]\right\}$$

$$+ \frac{Gr}{c^2(1-\mathrm{Pr})}\left\{\frac{\exp(ct)}{2}\left[\begin{array}{l}\exp\left(-2\eta\sqrt{\mathrm{Pr}(b+c)t}\right).erfc\left(\eta\sqrt{\mathrm{Pr}}-\sqrt{(b+c)t}\right)\\ +\exp\left(2\eta\sqrt{\mathrm{Pr}(b+c)t}\right).erfc\left(\eta\sqrt{\mathrm{Pr}}+\sqrt{(b+c)t}\right)\end{array}\right]\right\}$$

where, $b = \dfrac{R}{Pr}$, $c = \dfrac{R}{1-Pr}$, $d = \dfrac{Gr}{2c(1-Pr)}$, $e = \dfrac{Gct}{(1-Sc)}$ and $\eta = \dfrac{Y}{2\sqrt{t}}$.

RESULTS AND DISCUSSION

The numerical values of the velocity, temperature and concentartion are computed for different physical parameters like thermal radiation parameter, chemical reaction parameter, Schmidt number, thermal Grashof number and mass Grashof number. The value of the Schmidt number Sc is taken to be 0.6 which corresponds to water-vapour. Also, the value of Prandtl number Pr are chosen such that they represent air($Pr = 0.71$). The purpose of the calculations given here is to assess the effects of the parameters a, R, K, Gr, Gc and Sc upon the nature of the flow and transport. The solutions are in terms of exponential and complementary error function.

The temperature profiles for different values of thermal radiation parameter($R = 0.2, 2, 5, 10$), in the presence of air at time $t = 0.4$ are shown in figure 1. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. This shows there is a drop in temperature due to higher thermal radiation.

Figure 2, demonstrates the effect of temperature profiles for different values of time $(t = 0.2, 0.4, 0.6, 1)$ and $R = 0.2$ are shown in figure 2. It is observed that the wall temperature increases with increasing values of t.

The concentartion profiles for different values of the Schmidt number $(Sc = 0.16, 0.3, 0.6, 2.01)$ and $K = 0.2$ at time $t = 0.2$ are shown in figure 3. The effect of Schmidt number is important in concentration field. As expected, the concentration increases with decreasing values of the Schmidt number. The numerical values of the Schmidt number and the corresponding species are listed in the following table:

Species	Schmidt number	Name of the chemical species
H_2	0.16	Hydrogen
He	0.3	Heleum
H_2O	0.6	Water Vapour
$C_6H_5CH_2CH_3$	2.01	Ethyl Benzene

Figure 4, illustrates the effect of the concentration profiles for different values of the chemical reaction parameter $(K = 0.2, 2, 5, 10)$ at $t = 0.2$. The effect of chemical reaction parameter is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the concentartion increases with decreasing chemical reaction parameter.

The effect of the velocity for different values of the radiation parameter $R = 0.2, 5, 20), a = 0.9, K = 0.2, Gr = Gc = 5$ and $t = 0.4$ are shown in figure 5. The trend shows that the velocity increases with decreasing radiation parameter.

Figure 6, illustrates the effect of the velocity for different values of the chemical reaction parameter ($K = 0.2, 5, 10), R = 10, Gr = 5, Gc = 5, a = 0.9$ and $t = 0.4$. The trend shows that the velocity increases with decreasing chemical reaction parameter. It is observed that the velocity decreases in the presence of high thermal radiation.

The velocity profiles for different time ($t = 0.2, 0.3, 0.4), R = 0.2, a = 0.9, Gr = Gc = 2$ and $K = 0.2$ are shown in Figure 7. This shows that the velocity increases gradually with respect to time t.

Figure 8 demonstrates the effect of the velocity profiles for different values of thermal Grashof number $(Gr = 5, 10)$ and mass Grashof number $(Gc = 5, 10), K = 10, a = 0.9, R = 0.2$ and $t = 0.4$. It is clear that the velocity increases with increasing thermal Grashof number or mass Garshof number.

The velocity profiles for different ($a = 0.2, 0.5, 0.9$) Gr=Gc=5 and R=0.2 at $t = 0.4$ are studied and presented in figure 9. It is observed that the velocity increases with increasing values of a.

CONCLUSION
The theoretical solution of flow past an exponentially starting motion of the infinite vertical plate in the presence of variable temperature and uniform mass diffusion has been studied.The dimensionless governing equations are solved by the usual Laplace transform technique. The effect of different physical parameters like thermal radiation parameter, thermal Grashof number and mass Grashof number are studied graphically. The conclusions of the study are as follows:

i)   The velocity increases with increasing thermal Grashof number or mass Grashof number, but the trend is just reversed with respect to the thermal radiation parameter.
ii) The temperature of the plate increases with decreasing values of the thermal radiation parameter.

iii) The plate concentration increases with decreasing values of the Schmidt number.

REFERENCES

[1] P.L. CHAMBRE, J.D. YOUNG and Young JD (1958) On the diffusion of a chemically reactive species in a laminar boundary layer flow, The Physics of Fluids, 1, 48-54, 1958.

[2] U.N. DAS, R.K. DEKA, V.M. SOUNDALGEKAR, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forschung im Ingenieurwesen, 60, 284-287, 1994.

[3] U.N. DAS, R.K. DEKA, V.M. SOUNDALGEKAR, Effects of mass transfer on flow past an impulsively started infinite vertical plate with chemical reaction, The Bulletin, GUMA, 5,13-20, 1999.

[4] W.G. ENGLAND, A.F. EMERY, Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas, J Heat Transfer, 91, 37-44, 1969.

[5] M A. HOSSAIN, H.S. TAKHAR, Radiation effect on mixed convection along a vertical plate with uniform surface temperature, Heat and Mass Transfer, 31, 243-248, 1996.

[6] U.N. DAS, R.K. DEKA, V.M. SOUNDALGEKAR, Radiation effects on flow past an impulsively started vertical infinite plate, J Theo. Mech., 1, 111-115, 1996.

[7] A.S. GUPTA , I. POP, V.M. SOUNDALGEKAR, Convection effects on the flow past an accelerated vertical plate in an incompressible dissipative fluid, Rev. Roum. Sci. Techn.-Mec. Apl., 24, 561-568, 1979.

[8] N.G. KAFOUSIAS, A.A. RAPTIS, Mass transfer and free convection effects on The flow past an accelerated vertical infinite plate with variable suction or Injection, Rev.Roum. Sci. Techn.-Mec. Apl., 26,11-22, 1981.

[9] A.K. SINGH, NAVEEN KUMAR, Free convection flow past an exponentially accelerated vertical plate, Astrophysics and Space Science, 98, 245-258, 1984.

[10] M.A. HOSSAIN, L.K. SHAYO, The skin friction in the unsteady free convection flow past an accelerated plate, Astrophysics and Space Science, 125, 315-324, 1986.

[11] BASANT KUMAR JHA, RAVINDRA PRASAD, SURENDRA RAI, Mass transfer effects on the flow past an exponentially accelerated vertical plate with constant heat flux, Astrophysics and Space Science,.181,125-134, 1991.

NOMENCLATURE

A Constant

C' species concentration in the fluid $kg\,m^{-3}$

C dimensionless concentration

C_p specific heat at constant pressure $J.kg^{-1}.k$

D mass diffusion coefficient $m^2.s^{-1}$

Gc mass Grashof number

Gr thermal Grashof number

g acceleration due to gravity $m.s^{-2}$

k thermal conductivity $W.m^{-1}.K^{-1}$

Pr Prandtl number

Sc Schmidt number

T temperature of the fluid near the plate K

t' time s

u' velocity of the fluid in the x'-direction $m.s^{-1}$

u_0 velocity of the plate $m.s^{-1}$

u dimensionless velocity

y coordinate axis normal to the plate m

Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion K^{-1}

β^* volumetric coefficient of expansion with concentration K^{-1}

μ coefficient of viscosity $Ra.s$

ν kinematic viscosity $m^2.s^{-1}$

ρ density of the fluid $kg.m^{-3}$

τ dimensionless skin-friction $kg.m^{-1}.s^2$

θ dimensionless temperature

η similarity parameter

$erfc$ complementary error function

Subscripts

w conditions at the wall

∞ free stream conditions
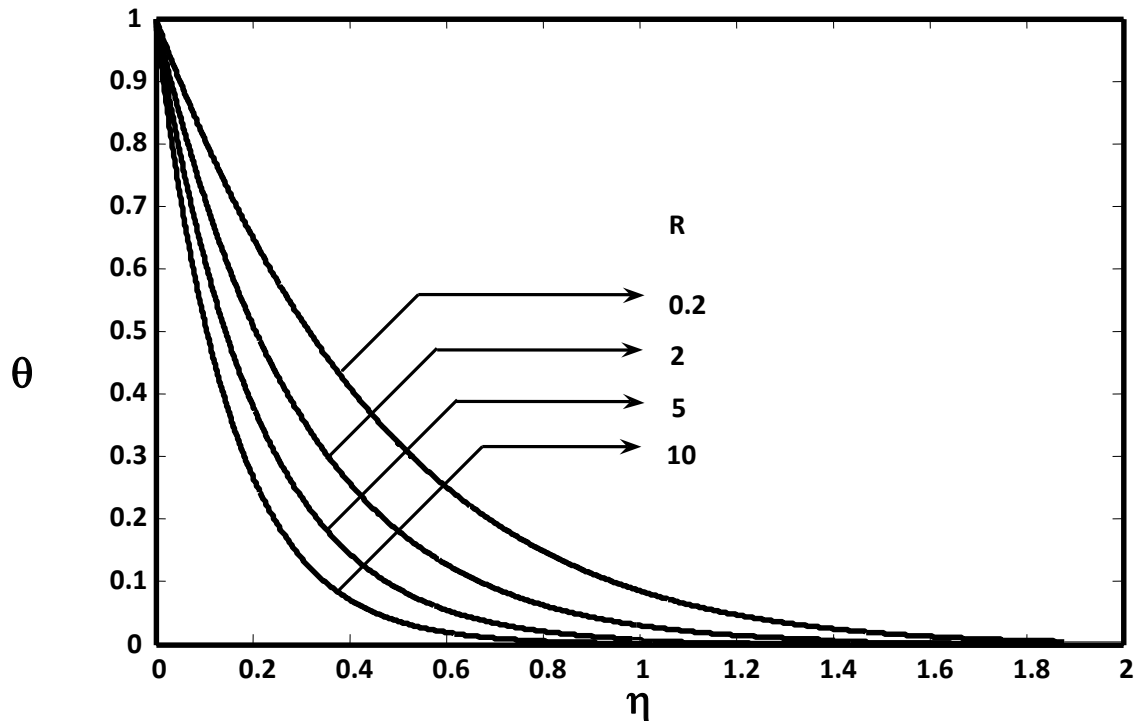
Figure 1. Temperature Profiles for different values of R
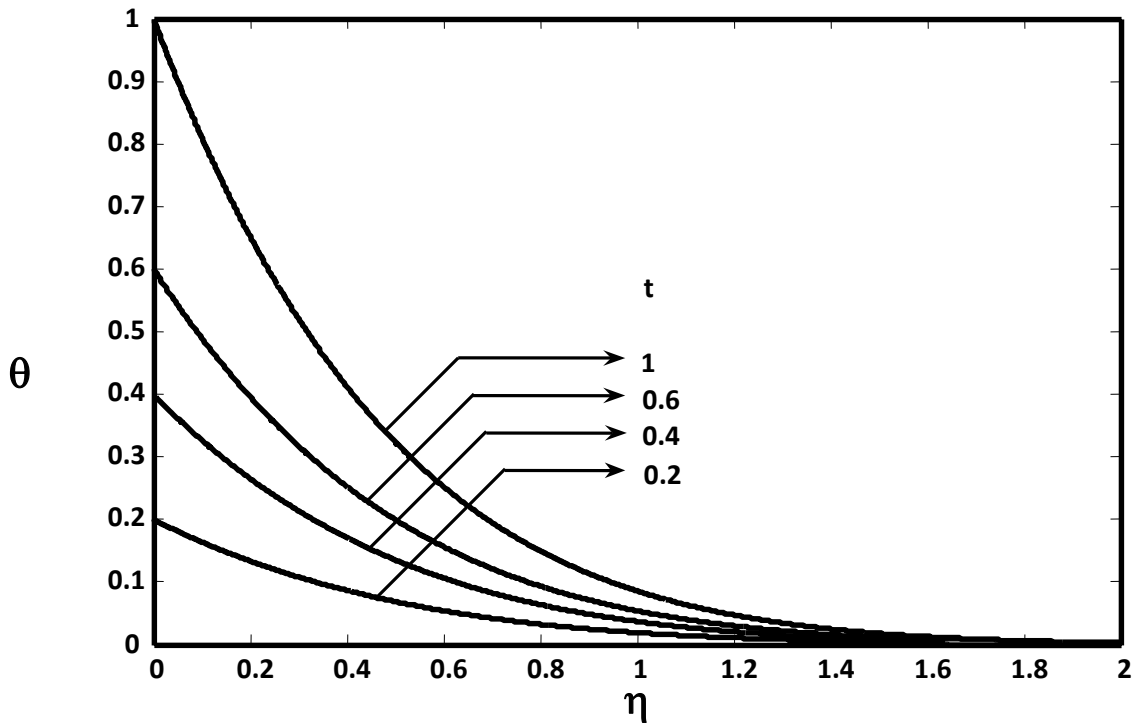
Figure 2. Temperature Profiles for different values of t
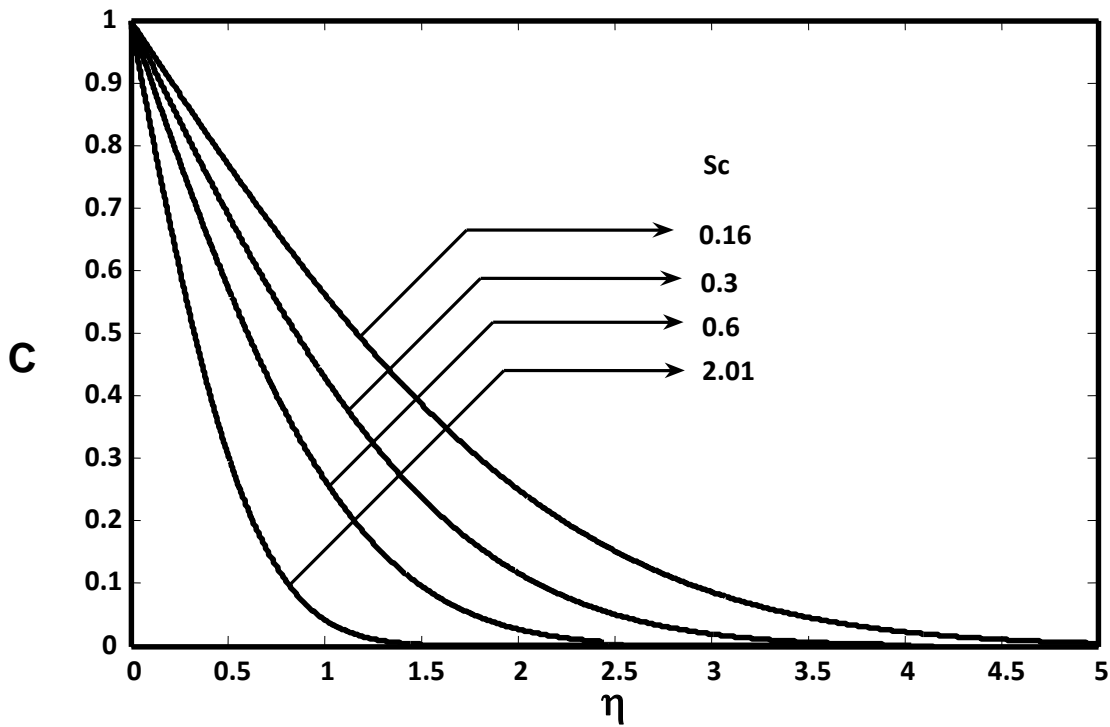
Figure 3. Concentration profiles for different values of Sc
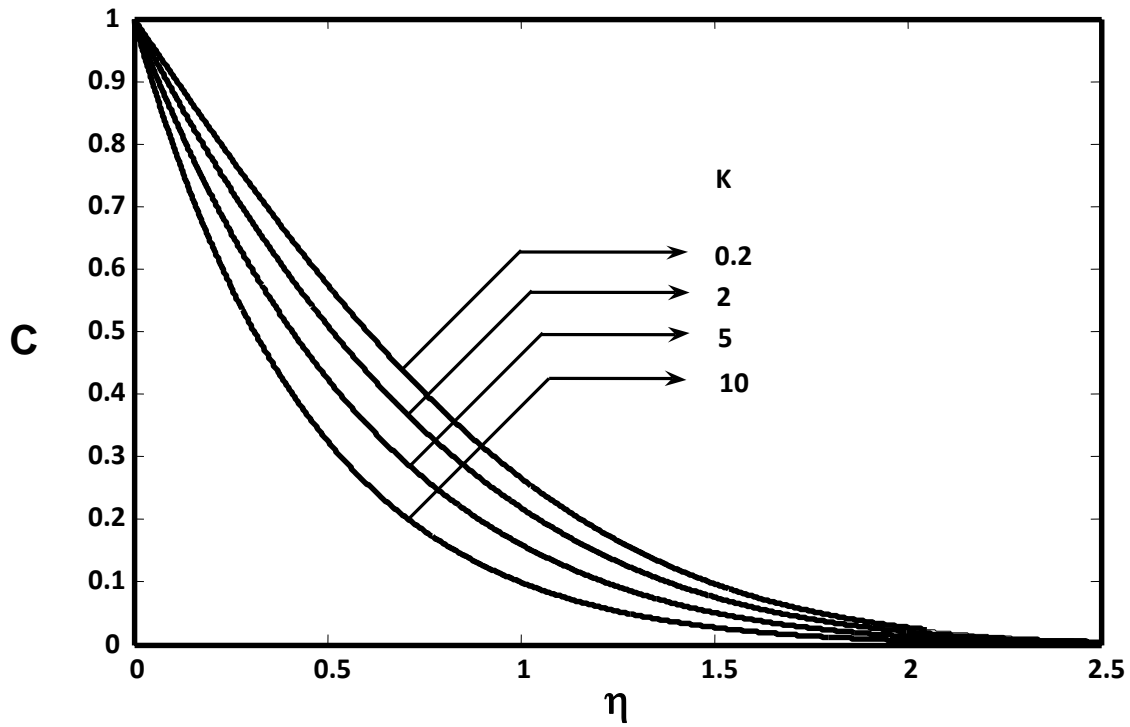
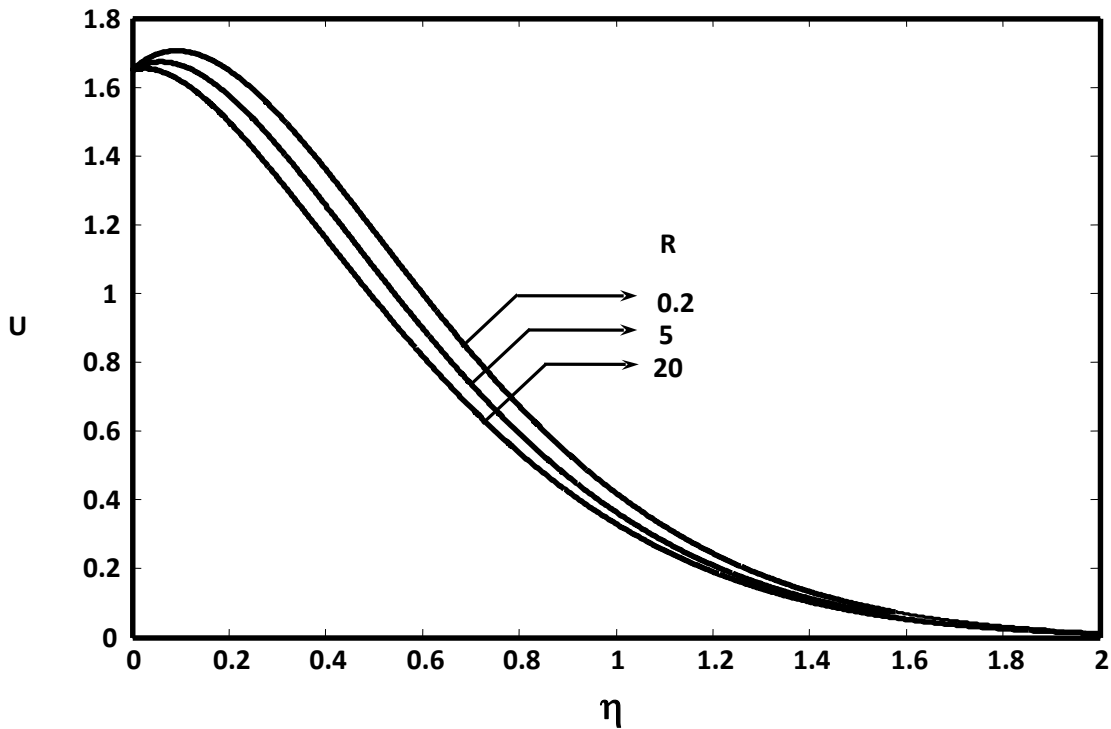*Figure 4. Concentration profiles for different values of K*

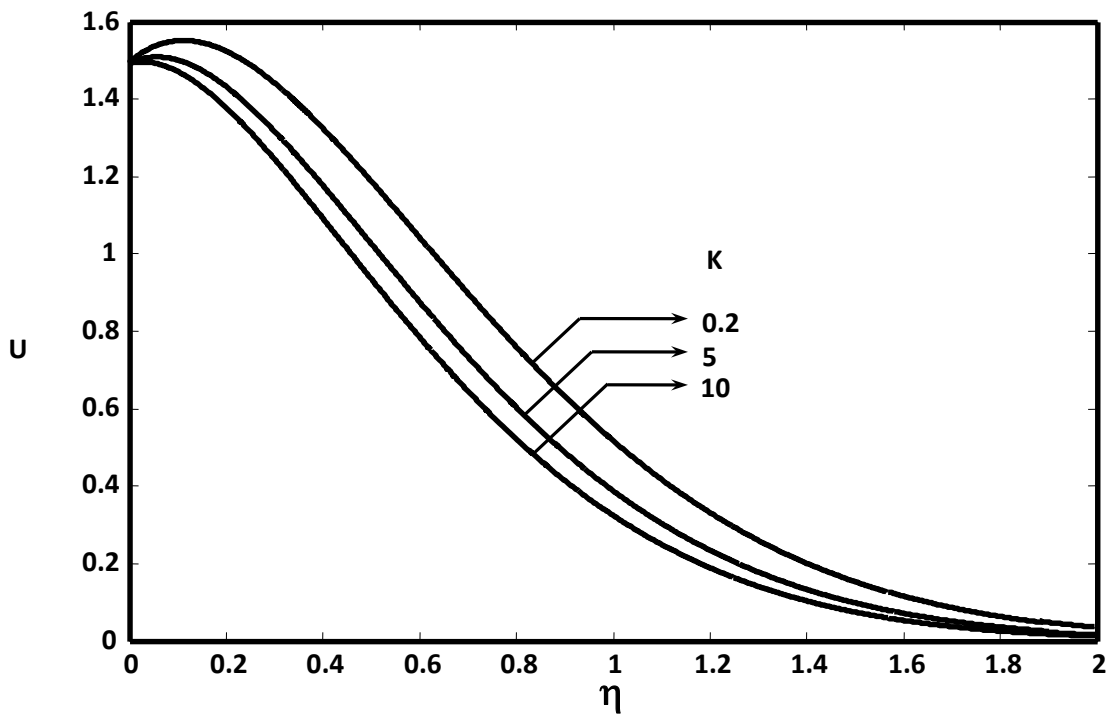*Figure 5.Velocity profiles for different values of R*

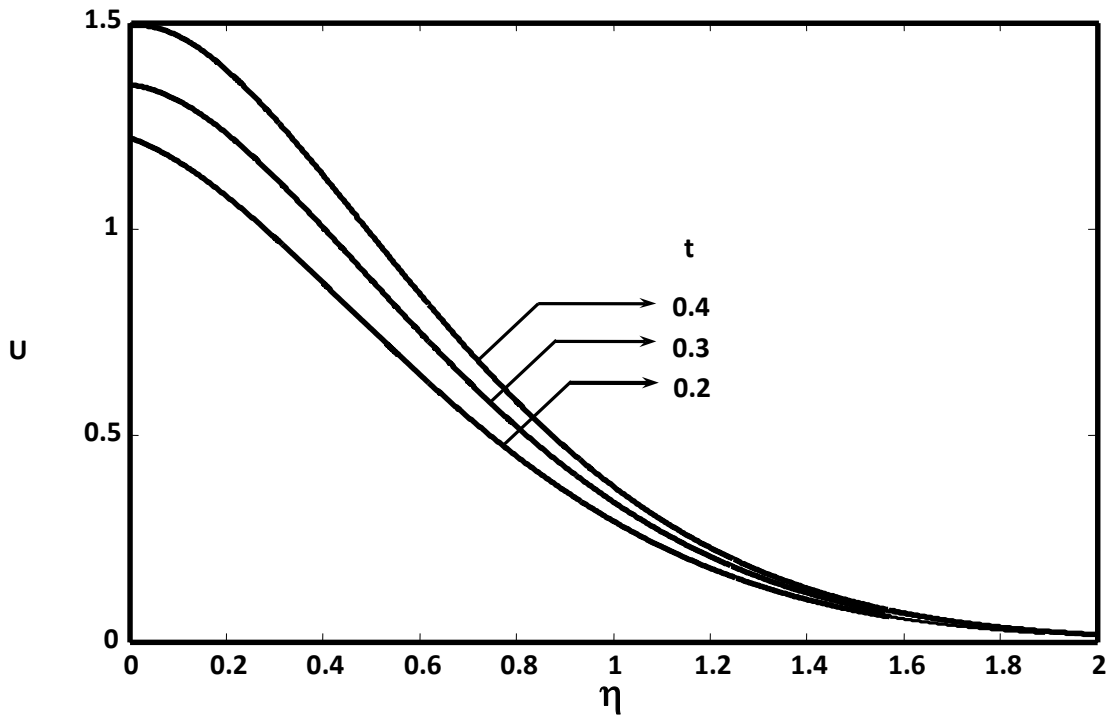*Figure 6.Velocity profiles for different values of K*

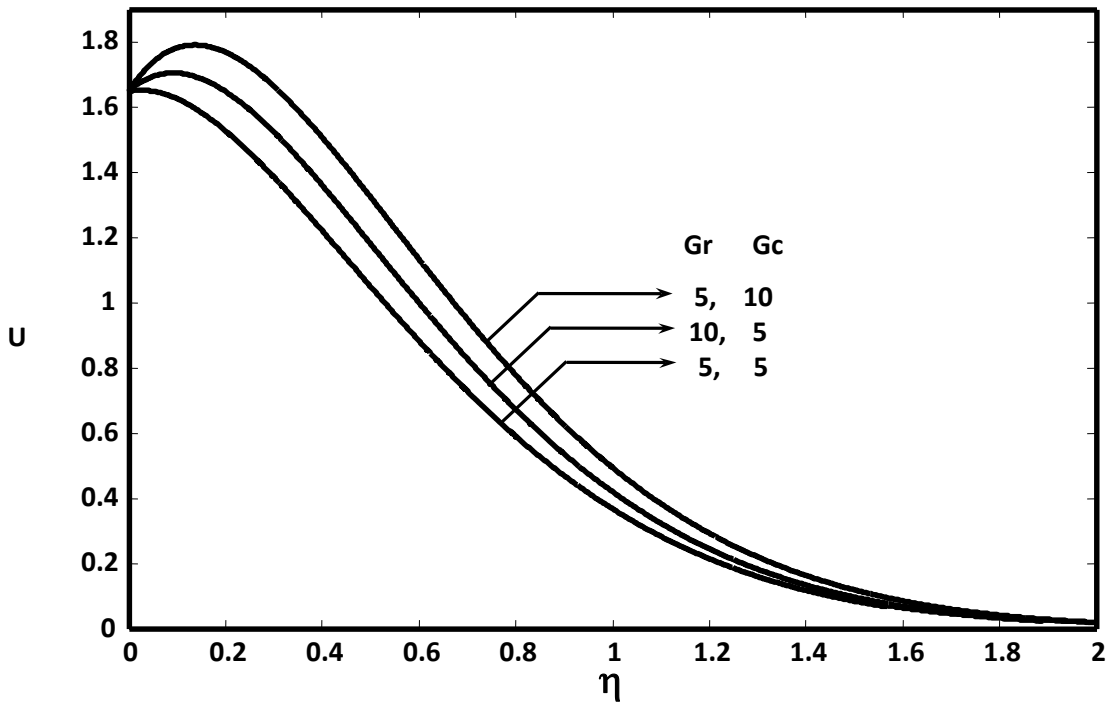*Figure 7.Velocity profiles for different values of t*
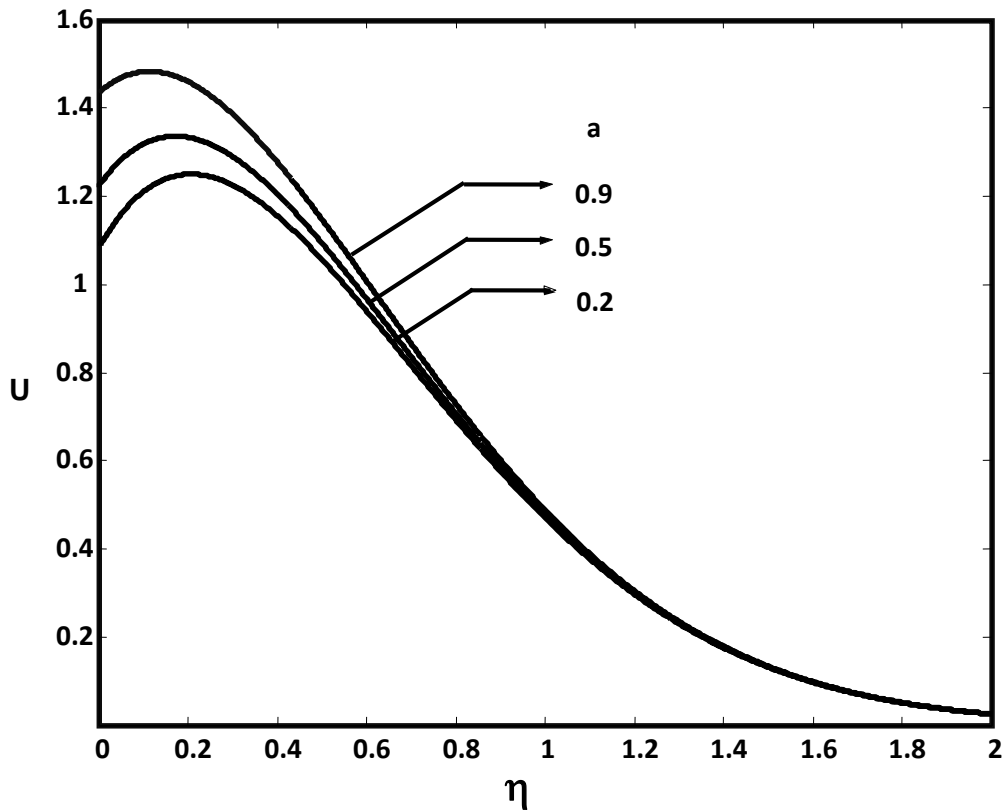
Figure 8.Velocity profiles for different values of Gr, Gc

Figure 9. Velocity profiles for different values of a